JOURNAL OF OPTIMIZATION THEORY AND APPLICATIONS: Vol. 128, No. 2, pp. 279–294, February 2006 (© 2006)
DOI: 10.1007/s10957-006-9016-9

Dynamic Cost-Benefit Analysis of Drug Substitution Programs[1]

J. P. CAULKINS,[2] G. FEICHTINGER,[3] C. GAVRILA,[4] A. GREINER,[5]
J. L. HAUNSCHMIED,[6] P. M. KORT,[7] AND G. TRAGLER[8]

Published Online: 2 June 2006

Abstract. This paper studies optimal spending for drug substitution programs in the context of a dynamic epidemic model of both drug use and drug use-related infections. Two types of costs are considered in addition to control costs: social costs resulting from individuals being dependent on drugs; additional costs due to drug users being vulnerable to infections like hepatitis C or HIV. Analysis of the model demonstrates that the long-run equilibrium is not necessarily unique. Instead, there may be multiple equilibria. Which of these equilibria is optimal depends on the initial conditions for the number of drug addicts and the number of those who are infected. So, for a given set of epidemic parameters, it may be optimal to spend a lot on substitution programs that reduce the number of drug addicts or to spend little and to accept a high level of drug use.

Key Words. Optimal control, drug prevention, thresholds, dynamic cost-benefit analysis.

---

[1] This research was financed in part by the Austrian National Bank (ÖNB) under Grant 9414 and by the Austrian Science Fund (FWF) under Grant P14060-G05. The authors are grateful to two anonymous referees for helpful comments.

[2] Professor, H. John Heinz III School of Public Policy and Management, Carnegie Mellon University, Pittsburgh, Pennsylvania.

[3] Professor, Institute for Mathematical Methods in Economics, Research Unit for Operations Research and Nonlinear Dynamical Systems, Vienna University of Technology, Vienna, Austria.

[4] Researcher, Dipartimento di Matematica, University of Rome – Tor Vergata, Rome, Italy.

[5] Professor, Department of Economics, Bielefeld University, Bielefeld, Germany.

[6] Assistant Professor, Institute for Mathematical Methods in Economics, Research Unit for Operations Research and Nonlinear Dynamical Systems, Vienna University of Technology, Vienna, Austria.

[7] Professor, Department of Econometrics, Tilburg University, Tilburg, Netherlands.

[8] Associate Professor, Institute for Mathematical Methods in Economics, Research Unit for Operations Research and Nonlinear Dynamical Systems, Vienna University of Technology, Vienna, Austria.

1. Introduction

Drug addiction imposes large costs on society. For the United States, Tami et al. (Ref. 1) estimate the total cost of heroin addiction in 1996 to be USD 21.9 billion. For France, Kopp et al. (Ref. 2) calculate the direct costs of drug addiction at EUR 0.73 billion. These numbers suggest that drug treatment is necessary not only to help those who are addicted but also to reduce the economic costs for society generally. Therefore, cost-benefit analyses should be conducted which determine the optimal amount to be spent for drug treatment. In this paper, we set up and analyze an optimal control model in order to determine the optimal spending level for treatment of drug dependence in a dynamic perspective that considers explicitly the spread of both drug use and infectious disease associated with that use. Most prior work on infectious disease spread among drug users has assumed that the number of drug users is a fixed constant and/or has not allowed the intensity of the treatment intervention to vary over time. Drug injection is the most common way that such diseases are spread and heroin is the most commonly injected drug, so we couch the analysis in terms of substitute therapy (e.g. methadone maintenance) for heroin dependence, but the framework might also be relevant for other injected drugs (e.g. methamphetamine) or other vectors of disease transmission.

In order to determine how much a social planner should spend for treatment, first one has to identify the costs of drug addiction. One type of costs are those resulting from drug purchase and use, notably criminal activities of drug addicts and their suppliers and productivity losses due to the fact that drug addicts do not contribute as much to the GDP. A distinct category of costs, especially associated with injection drug users (IDUs), results from the fact that drug use can spread costly infectious diseases such as hepatitis C (HCV), human immunodeficiency virus (HIV) infections, and tuberculosis to mention just three. Parts but not all of these costs[9] can be avoided by treating the addicts.

A complete cost-benefit analysis would calculate total costs of treating drug addicts and compare these to the benefits resulting from preventing all costs which would occur without intervention. However, a simultaneous treatment of all possible medical complications from drug addiction in an optimal control model would be cumbersome and might or might not yield additional structural insights. Therefore, generally one confines the analysis at least initially to one type of medical complication associated with drug addiction (see e.g. Refs. 3–4).

In this paper, we consider the case of HCV infections, since injecting drug use is a major risk for HCV infections and HCV is widespread among IDUs.

[9] For example, it cannot be assumed that addicted users receiving treatment are as productive as people not suffering from drug addiction.

The rest of the paper is organized as follows. In Section 2, we present the control model. Section 3 derives the dynamic system: first we study the analytical model and then present a simulation study with empirically relevant parameters. Section 4 gives an interpretation of the results. Finally, Section 5 concludes the paper.

2. Model

We start the description of our model with the differential equation describing the evolution of the number of IDUs. This equation is given by

$$\dot{N}(t) = \theta - \delta(N(t) - M(t)) - \mu\gamma M(t), \quad N(0) = N_0, \tag{1}$$

with[10] N representing the number of IDUs and M the number of addicts who get methadone treatment. $\theta, \delta, \mu, \gamma$ are exogenous parameters with the following interpretation: θ gives the exogenous increase in the number of IDUs due to initiation or immigration; δ is the per capita outflow of IDUs who do not receive methadone treatment, including outmigration, death, and quitting without the assistance of the treatment control intervention considered here; μ gives the exit rate from IDUs due to methadone treatment; γ is the proportion of those exiting who do not relapse.

The treatment community often rejects terminology of "curing" addiction, which is seen as a chronic relapsing disease, but for those outside the medical community it may be a useful shorthand to think of this as a "cure rate." Defining $U \equiv M/N \in (0, 1)$ and $a \equiv \mu\gamma - \delta$, equation (1) can be rewritten as

$$\dot{N} = \theta - \delta N - aUN, \quad N(0) = N_0. \tag{2}$$

A certain proportion of IDUs are infected by HCV. Infection among IDUs is spread by sharing injection equipment (syringes, cookers, etc.), which we refer to in shorthand as "needles." As in Ref. 3, the number of infected IDUs, denoted by I, evolves according to the following differential equation:

$$\begin{aligned}\dot{I} = {} & \kappa\psi(1 - M/N)(\Omega N - 1)[I/(\Omega N)] \\ & - \delta I(1 - M/N) - \mu\gamma M(I/N), \quad I(0) = I_0.\end{aligned} \tag{3}$$

The parameter Ω denotes the fraction of IDUs who engage in needle sharing. Those who do not share are not at risk of infection, but they do "absorb" treatment slots and impose costs on society due to their drug use. The parameter ψ is the frequency with which IDUs share needles. In the literature, this is sometimes called visiting "shooting galleries," places where IDUs take drugs and randomly mix with others present at these locations. There are other types of sharing with

[10] In the following, we omit the time argument t.

variation across regions in which modalities predominate, but such details are not essential for the present purposes. The probability of infection in the case of needle sharing is given by κ.

Thus, the terms on the right-hand side of (3) have the following interpretations. The first term gives the frequency with which noninfected IDUs who do not receive methadone treatment go to shooting galleries, $\psi(1 - M/N)(\Omega N - I)$, times the probability of meeting an infected IDU there, $I/(\Omega N)$, times the probability of infection given such a meeting κ. This term gives the gross increase in the number of infected IDUs per time period. The second term on the right-hand side gives the outflow of infected IDUs without intervention; the last term is the incremental decline in infected IDUs due to methadone treatment. The sum of these three terms describes the total change in the number of infected IDUs per time period.

Defining $b \equiv \kappa\psi$, $f \equiv \kappa\psi/\Omega$ and recalling the definition of U and a, we can rewrite equation (3) as follows:

$$\dot{I} = I[(1 - U)(b - fI/N) - \delta - aU], \quad I(0) = I_0. \tag{4}$$

As is obvious from equations (2) and (4), both the total number of IDUs and the number of infected IDUs are reduced by the methadone treatment.

As to the costs of the methadone treatment, we assume that they are given by

$$C(M) = \alpha M + \beta M^2,$$

with $\alpha, \beta > 0$, or using $U = M/N$, by

$$C(NU) = \alpha NU + \beta(NU)^2.$$

The assumption of a quadratic cost function reflects two types of diminishing returns. First, there is heterogeneity across IDUs in the treatment effectiveness, and the easier to treat are usually treated first ("cream skimming"). So, for every additional IDU treated, more money needs to be spent on methadone treatment in order to achieve the same result. Second, it is more difficult to motivate an additional IDU to go into methadone treatment if the group of people getting treatment is already large. The unit costs of an IDU, N, are given by ρ and the additional costs of an infected IDU, I, are denoted by ν. It should be noted that the number of IDUs is composed of both infected and noninfected IDUs. Thus, our formulation of the costs implies that we can identify the additional costs generated by an IDU being infected.

Examples of costs due to current drug use include costs from drug the addicts criminal activities and reduced productivity, as mentioned already in the Introduction. These costs are best modeled as a constant cost per active user. In contrast, the present value of all future costs of treating an HCV infection are charged in the objective function at the time of infection, because the future trajectory of costs is largely independent on whether the individual is or is not in methadone treatment.

Furthermore, society would continue to pay those costs of treating HCV even if the individual stopped using drugs and hence left the model. It should also be pointed out that only the averted costs of addicts not becoming infected due to methadone treatment is of relevance. This holds because infected people at any time will remain infected and need treatment independently of whether or not they receive methadone treatment or indeed whether they continue to inject drugs. If infected people receive methadone treatment, only the costs due to their being addicted to drugs can be avoided. Thus, there is a qualitative difference in the costs caused by somebody being infected and by a noninfected drug addict.

The optimization problem then is to design a methadone program intensity over time such that the total cost is minimized subject to the equations giving the evolution of the total number of IDUs and of the number of infected IDUs. These consist of the costs for the program itself, the costs of people being drug dependent, and the costs of new infections, which can be avoided by methadone programs. Denoting with r the planner intertemporal discount rate, the optimization problem is formally written as

$$\max_{U\in(0,1)} \int_0^\infty \exp(-rt)\big[-\alpha NU - \beta(NU)^2 - \rho N - \nu I(b - fI/N)(1-U)\big]dt, \tag{5}$$

subject to (2) and (4).

In the following, we define the new variable

$$P = I/N.$$

The dynamics of P then is given by

$$\dot{P} = P(\dot{I}/I - \dot{N}/N).$$

This gives

$$\dot{P} = P[(1-U)(b - fP) - \theta/N], \quad P(0) = I_0/N_0. \tag{6}$$

The optimization problem then is rewritten as

$$\max_{U\in(0,1)} \int_0^\infty \exp(-rt)\big[-\alpha NU - \beta(NU)^2 - \rho N - \nu PN(b - fP)(1-U)\big]dt, \tag{7}$$

subject to (2) and (6).

To solve this problem, we formulate the current-value Hamiltonian

$$H(\cdot) = -\nu PN(b - fP)(1-U) - N\rho - \alpha NU - \beta N^2U^2 + \lambda_1(-\delta N + \theta - aNU) + \lambda_2 P[(1-U)(b - fP) - \theta/N]. \tag{8}$$

The necessary optimality conditions are given by

$$\partial H/\partial U = 0 \leftrightarrow U = -\big(\alpha N + a\lambda_1 N + b\lambda_2 P - bN\nu P - f\lambda_2 P^2 + fN\nu P^2\big)/\big(2\beta N^2\big), \tag{9}$$

$$\dot{\lambda}_1 = r\lambda_1 - \partial H/\partial N, \tag{10}$$

$$\dot{\lambda}_2 = r\lambda_2 - \partial H/\partial P. \tag{11}$$

Further, we posit that the limiting transversality condition,

$$\lim_{t\to\infty} \exp(-rt)(\lambda_1 N + \lambda_2 p) = 0,$$

must be fulfilled.

3. Dynamic Behavior

3.1. Dynamics of the Model with a Nonoptimal Control. Before we study the dynamics of the model with U chosen optimally, we analyze the two differential equations (2) and (6) for a constant and arbitrarily chosen U. With this assumption, there exist two solutions to $\dot{N} = 0$ and $\dot{P} = 0$ given by[11]

$$\bar{N} = \theta/(\delta + aU), \quad \bar{P}^1 = 0,$$

and by

$$\bar{N} = \theta/(\delta + aU), \quad \bar{P}^2 = [\delta - b + U(a + b)]/[f(U - 1)].$$

$\bar{P}^2 > 0$ can exist only for

$$U < (b - \delta)/(a + b).$$

This implies that the second stationary point exists only if b, the product of the probability of infection and the frequency with which IDUs come to shooting galleries, is smaller than the outflow of IDUs per time period. Or stated in another way, if the outflow of IDUs is sufficiently large, only the boundary solution $\bar{P}^1 = 0$ exists.

The eigenvalues of the Jacobian at $(\bar{N}, \bar{P}^1)$ are given by

$$\mu_1^1 = -\delta - aU \quad \text{and} \quad \mu_2^1 = -\delta - aU + b(1 - U).$$

This shows that, for $\delta > b$, both eigenvalues are negative. For $\delta < b$, both eigenvalues are negative if

$$U > (b - \delta)/(a + b),$$

[11] The bar denotes stationary values.

while one eigenvalue is positive and one is negative for

$$U < (b - \delta)/(a + b).$$

The eigenvalues at the second stationary point $(\bar{N}, \bar{P}^2)$ are obtained as

$$\mu_1^1 = -\delta - aU \quad \text{and} \quad \mu_2^1 = \delta + aU - b(1 - U).$$

Since

$$U < (b - \delta)/(a + b)$$

must hold for the second stationary point to be reasonable, both eigenvalues are negative at this stationary point.

The optimal control U depends on N, P and their shadow prices and affects the dynamics of the model. Thus, it is to be expected that a more complex dynamic behavior may be the outcome of the optimally controlled model.

3.2. Optimal Control, Analytical Results. Inserting (9) in equations (2), (6), (10), (11), we get an autonomous differential equation system describing the evolution of our model over time. This system is given by

$$\dot{N} = -\delta N + \theta - aNU(\cdot), \tag{12}$$

$$\dot{P} = P[(1 - U(\cdot))(b - fP) - \theta/N], \tag{13}$$

$$\begin{aligned}\dot{\lambda}_1 = {} & \lambda_1 r + \rho - \lambda_2 P\theta/N^2 + \nu\left(bP - fP^2\right)(1 - U(\cdot)) + \alpha U(\cdot) \\ & + 2\beta N U^2(\cdot) - \lambda_1(-\delta - aU(\cdot)),\end{aligned} \tag{14}$$

$$\begin{aligned}\dot{\lambda}_2 = {} & \lambda_2 r - \lambda_2[-\theta/N - fP(1 - U(\cdot)) + (b - fP)(1 - U(\cdot))] \\ & + \nu(bN - 2fNP)(1 - U(\cdot)),\end{aligned} \tag{15}$$

with $U(\cdot)$ given by (9). Stationary points are solutions to the system $\dot{N} = \dot{P} = \dot{\lambda}_1 = \dot{\lambda}_2 = 0$. To find such solutions, we proceed as follows. First, we set $\dot{\lambda}_1 = 0$ and solve this equation with respect to λ_1 giving $\lambda_1 = \lambda_1(N, P, \lambda_2, \cdot)$. Inserting $\lambda_1(N, P, \lambda_2, \cdot)$ in equation (12), setting $\dot{N} = 0$ and solving this equation with respect to λ_2 yields $\lambda_2 = \lambda_2(N, P, \cdot)$. Inserting $\lambda_1(N, P, \lambda_2(N, P, \cdot), \cdot)$ and $\lambda_2(N, P, \cdot)$ in equation (13) leads to an equation which depends only on N and P. Setting this equation equal to zero and solving for P yields two solutions given by

$$\bar{P}^1 = 0, \tag{16}$$

$$P(N, \cdot) = (abN + b\delta N - a\theta - b\theta)/[f(aN + \delta N - \theta)]. \tag{17}$$

A first stationary point is obtained by setting $P = 0$ and solving $\dot{N} = 0$ and $\dot{\lambda}_2 = 0$ with respect to λ_1 and λ_2 respectively, giving $\lambda_1 = \lambda_1(N, \cdot)$ and $\lambda_2 = \lambda_2(N, \cdot)$. Inserting $P = 0, \lambda_1 = \lambda_1(N, \cdot)$ and $\lambda_2 = \lambda_2(N, \cdot)$ in equation (14), setting that

equation equal to zero, and solving with respect to N gives $\bar{N}^1$ as a function of parameters alone. The stationary point E^1 then is obtained as $E^1 = (\bar{N}^1, \bar{P}^1, \bar{\lambda}_1^1, \bar{\lambda}_2^1)$, with

$$\bar{N}^1 = \{a[\alpha(\delta + r) - a\rho] + 2\beta(\delta + r)\theta\}/[2\beta\delta(\delta + r)], \tag{18}$$

$$\bar{P}^1 = 0, \tag{19}$$

$$\bar{\lambda}_1^1 = -\rho/(\delta + r), \tag{20}$$

$$\bar{\lambda}_2^1 = K_1/K_2, \tag{21a}$$

$$K_1 = b\nu[a\alpha(\delta + r) - a^2\rho + 2\beta(\delta + r)\theta] \cdot\{-a^2\rho + a[\alpha(\delta + r) - \delta\rho] + (\delta + r)(\alpha\delta + 2\beta\theta)\}, \tag{21b}$$

$$K_2 = 2\beta\delta(\delta + r)\{a^2(-b + r)\rho + a[\alpha(b - r)(\delta + r) - b\delta\rho] + (\delta + r)[\alpha b\delta + 2\beta(b - \delta - r)\theta]\}. \tag{21c}$$

It should be noted that λ_1 (the shadow price of N) is always negative while λ_2 (the shadow price of P) may be negative or positive. This holds because more IDUs implies higher costs, whereas the effect of the number of infected on total costs is not unambiguous. The latter is due to the fact that only the number of averted infections matters as outlined above.

The control at this stationary point $\bar{U}^1$ is obtained as

$$\bar{U}^1 = -\delta[\alpha(\delta + r) - a\rho]/\{a[\alpha(\delta + r) - a\rho] + 2\beta(\delta + r)\theta\}. \tag{22}$$

It should be noted that $\bar{U}^1 \in (0, 1)$. In order to get a positive value for $\bar{U}^1$, i.e. $\bar{U}^1 > 0, a\rho > \alpha(\delta + r)$ must hold.[12] This shows that, in order to eradicate HCV infection among IDUs, spending money for methadone in the long-run can only be optimal if the cost of methadone treatment α is low compared to the cost caused by drug addicts ρ and if the "cure rate" of treatment a is high. Intuitively, this is clear because it cannot be optimal from an economic point of view to reduce the number of infected IDUs to zero when the costs of treatment are high or the cure rate is small.

Other stationary points are obtained as follows. We insert

$$P(N, \cdot) = (abN + b\delta N - a\theta - b\theta)/[f(aN + \delta N - \theta)]$$

in equation (15). This gives a function $g(N, \cdot)$ which depends only on N and the parameters. Setting $g(N, \cdot) = 0$ and solving that equation with respect to N then gives the stationary points $\bar{N}$ as a function of the parameters alone. It should be noted that the solutions of $g(N, \cdot) = 0$ must be such that $\bar{P} \in (0, 1)$ is satisfied,

[12] Note that the denominator is positive. Otherwise, $\bar{N}^1$ would be negative.

in addition to $\bar{U}^1 \in (0, 1)$. However, the equation $g(N, \cdot) = 0$ cannot be solved analytically, but only with numerical values for the parameters.

To study the local dynamics, we calculate the Jacobian matrix at the stationary point under consideration, which in principle can be computed for the analytical model. However, doing so does not yield insights in the dynamics, since the matrix becomes very complicated. Therefore, we restrict our analysis in what follows to numerical examples with realistic parameter values.

What we can say as concerns the analytical model is that the model has at most a saddle point, i.e. the Jacobian cannot have only negative eigenvalues or eigenvalues with only negative real parts. This holds because the Jacobian matrix for that type of control models can be written as

$$J = \begin{pmatrix} H^0_{\lambda x} & H^0_{\lambda\lambda} \\ -H^0_{xx} & -H^0_{x\lambda} + rE \end{pmatrix},$$

with E being the identity matrix, H^0 is the maximized Hamiltonian, and $x = (N, P)$. The eigenvalues of that matrix are

$$\mu_1 = (r/2) + \bar{\xi} \quad \text{and} \quad \mu^1 = (r/2) - \bar{\xi},$$

where $\bar{\xi}$ is the eigenvalue of the matrix $F(r/2)$ (see Ref. 5, Theorem 2), that is given by

$$F(r/2) = \begin{pmatrix} H^0_{\lambda x} - (r/2)E & H^0_{\lambda\lambda} \\ -H^0_{xx} & -H^0_{x\lambda} + (r/2)E \end{pmatrix}.$$

3.3. Simulation Study. Next, we compute the solution for numerically specified parameter values which are empirically relevant[13] and where we consider one time period as one year. The computations were done with Mathematica, see Ref. 6.

The exit rate from the active IDU population δ is based on the average of those in the Kaplan "needles that kill" study (Ref. 7) and in the Thomas et al. report from the Baltimore ALIVE cohort (Ref. 8). In particular, Kaplan used a figure of $1/2920 = 3.42 \times 10^{-4}$; Pollack (Ref. 4), adjusting the ALIVE data for length-biased sampling, suggests $1/6319 = 1.58 \times 10^{-4}$. The average, $\delta = 2.5 \times 10^{-4}$ implies an average career length of 4000 days or about 11 years.

The inflow parameter θ linearly scales the population to the size of the city being modelled; as in Ref. 3, we take that to be a new user every other day ($\theta =$ 183 per year) implying that, in the absence of treatment, the steady state population

[13] Most of these values are taken directly from Refs. 3–4.

would be $\theta/\delta = 2000$. That means that, we are modelling a small city or a large neighbourhood within a city, though again, the results are independent of scale.

The exit rate due to treatment μ is 0.9125 and the permanent cure rate γ is 0.2. These parameters imply that the parameter a is equal to 0.09125.

Pollack (Ref. 4) considers a range of infectivities between $0.005 < \kappa < 0.05$ and concludes that those toward the upper end of the range better model the rapid spread of HCV, so we set $\kappa = 0.04$.

As has been customary in OR models of injection drug use since the Kaplan pioneering work (Ref. 7), we assume that the injection drug users visit shooting galleries once per week, so $\psi = 52$ per year.

As in Ref. 9, the proportion of shooting gallery participants Ω is 30 percent. These parameter values give $b = 2.08$, $f = 6.93$ for HCV.

The cost per year of one IDU is $\rho = 6000$. A reasonable assumption as to the lifetime costs of an HCV infection is between 7300 USD and 18250 USD implying for ν about $\nu \in [1000, 13000]$; cf. Ref. 10. We set in our simulations $\nu = 8000$ as benchmark. The costs of the methadone program are 4015 USD per patient per year. We assume a convex cost function and set $\alpha = 4014$ and $\beta = 1$ implying $C(1) = 4015$. As is customary in medical cost-effectiveness analysis, we set the annual discount rate to $r = 0.03$ (Ref. 11).

3.3.1 Boundary Stationary Solution $\bar{\mathrm{P}} = 0$. The first stationary point $E^1 = (\bar{N}^1, \bar{P}^1, \bar{\lambda}_1^1, \bar{\lambda}_2^1)$ is given by $E^1 = (1754.747, 0, -49484.536, 1.5110^7)$. Here and throughout, results are stated with high precision to make it easier for others to replicate them, but it should be understood that simplifications inherent in the modeling and imprecision in the parameter value estimates imply that the solutions for a substantive perspective do not have this degree of precision. The control at the stationary point is $\bar{U}^1 = 0.143$. The eigenvalues associated with this stationary point are

$$\mu_1^1 = -1.648505, \quad \mu_2^1 = -0.09125, \quad \mu_3^1 = 0.12125, \quad \mu_4^1 = 1.678505.$$

This shows that the stationary point is a saddle point.

Varying the parameter ν between $\nu = 1000$ and $\nu = 14000$ with a stepsize of 1000 does not change the qualitative outcome.[14]

3.3.1. Interior Stationary Solutions. Solving $g(N, \cdot) = 0$ yields three feasible interior stationary solutions,[15] namely:

$$E^2 = (\bar{N}^2, \bar{P}^2, \bar{\lambda}_1^2, \bar{\lambda}_2^2)$$
$$= (1048.022, 0.009, -56136.066, -4.04510^7), \quad \text{with } \bar{U}^2 = 0.914,$$

[14] This was to be expected, since $\bar{\lambda}_2^1$ depends only on ν.

[15] There is a fourth interior solution which, however, is not feasible, since it implies $\bar{U} > 1$.

$$
\begin{aligned}
E^3 &= (\bar{N}^3, \bar{P}^3, \bar{\lambda}_1^3, \bar{\lambda}_2^3) \\
&= (1122.435, 0.19, -55435.717, 3.39810^6), \quad \text{with } \bar{U}^3 = 0.787, \\
E^4 &= (\bar{N}^4, \bar{P}^4, \bar{\lambda}_1^4, \bar{\lambda}_2^4) \\
&= (1734.813, 0.282, -49672.157, 1.29610^7), \quad \text{with } \bar{U}^4 = 0.156.
\end{aligned}
$$

The eigenvalues $\mu_{1,2,3,4}$ of the Jacobian matrix associated with these stationary points are given by

$$
\begin{aligned}
&(E^2) \quad \mu_{1,2}^2 = 0.158241 \pm 0.0951999i, \quad \mu_{3,4}^2 = -0.128241 \pm 0.0951999i, \\
&(E^3) \quad \mu_{1,2}^3 = 0.015 \pm 0.437164i, \quad \mu_3^3 = 0.225219, \mu_4^3 = -0.195219, \\
&(E^4) \quad \mu_1^4 = 1.66122, \mu_2^4 = -1.63122, \quad \mu_3^4 = 0.119684, \mu_4^4 = -0.089684.
\end{aligned}
$$

This implies that the second stationary point E^2 is a saddle point with complex conjugate eigenvalues. The third stationary point E^3 is unstable with the exception of a one-dimensional stable manifold; the fourth stationary point E^4 is a saddle point with real eigenvalues.

Again, we varied the parameter ν between $\nu = 1000$ and $\nu = 14000$. For all $\nu \in [2873, 14000]$, we get the same outcome from a qualitative point of view, i.e. three feasible interior solutions, with two being saddle point stable and one being unstable. For ν smaller than 2873, however, only the fourth stationary point is a feasible interior solution, while the other two stationary points do not yield plausible values but are complex conjugate numbers. Further, this stationary point is a saddle point with two negative and two positive real eigenvalues.

The result that there are two saddle point stable stationary points and one unstable stationary point for $\nu \in [2873, 14000]$ suggests that there exists a threshold or DNS (Dechert-Nishimura-Skiba) curve[16] which separates the basins of attraction for optimal paths. This means that the initial conditions are crucial as to whether it is optimal to converge to E^2 or E^4. Only if the initial values of N and P are on the DNS curve is the planner indifferent between the two stationary points, because they yield the same value for (7). In all other cases, only convergence to one of the two saddle points is cost minimizing.

To demonstrate that a DNS curve does exist in our model, we fix the parameter ν and set it to $\nu = 12000$. Figure 1 shows the resulting paths leading to the equilibria E^2 and E^4 in the (N, P) space.

The DNS curve splits the state space into two regions of optimal convergence. If the starting values of infected per IDUs, $P(0)$, and of IDUs, $N(0)$, lie in the grey shaded area, it is optimal to converge to equilibrium $E^2 = (\bar{N}^2, \bar{P}^2) = (1048, 0.009)$. If the starting values lie in the white shaded area, then it is optimal to converge to equilibrium $E^4 = (\bar{N}^4, \bar{P}^4) = (1734.8, 0.282)$.

[16] Named after Dechert and Nishimura (Ref. 12) and Skiba (Ref. 13), who did seminal work within this field.

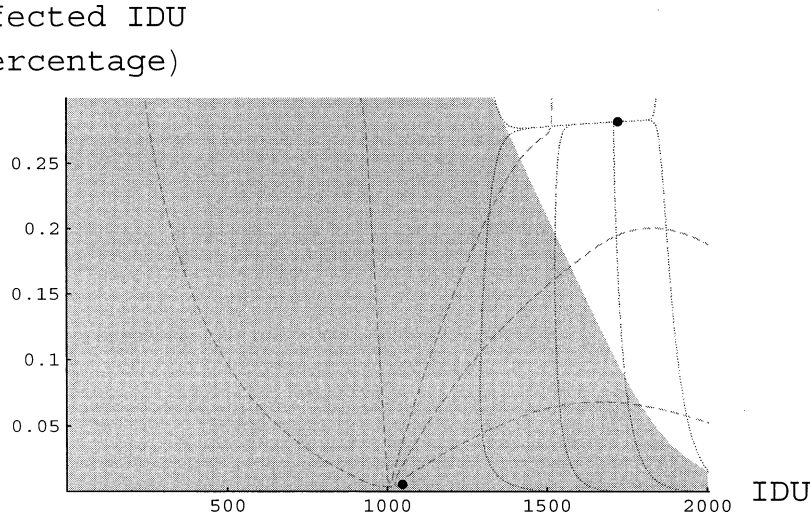

Fig. 1. Picture of the global dynamics with parameter values specified in Section 3.3. The DNS curve is the border between the grey shaded area [convergence to $E^2 = (1048, 0.009)$] and the white shaded area [convergence to $E^4 = (1734, 0.282)$].

Figure 1 gives a picture of the global dynamics with parameter values as specified above. Another interesting question is for which values of ν, which determine the cost of a new HCV infection, does such a threshold exists. In our example, it turns out that this holds for $\nu \in [9355.4, 13253.6]$. If ν is smaller than 9355.4, no DNS curve exists and the only equilibrium is E^4, where both the number of infected and noninfected IDUs is high.[17] If ν is larger than 13253.6, it is always optimal to spend a lot for methadone treatment and to reduce the number of infected IDUs to a great degree. In this case, again no DNS curve exists and the only feasible equilibrium is E^2.

It should be mentioned also that the domain of ν for which a threshold exists depends also on the infectivity κ. The larger the infectivity, the larger is the interval of ν for which a DNS curve exists. For example, for $\kappa = 0.08$, a DNS curve exists for $\nu \in [9168, 13391]$. Further, with a rising κ, the DNS curve moves up and to the right.

Not surprisingly, whether or not a DNS curve exists depends also on the value of the discount rate r. Above, we showed that, for the given parameter values, a DNS curve exists if $r = 0.03$. Leaving all other parameters at their base case values (see beginning of Section 3.3), we find that a DNS curve arises for $r \in [0.016, 0.125]$, separating the equilibria with high and low levels of the

[17] The exact numerical values of the equilibrium depend of course on the value of ν.

control, respectively. However, if $r < 0.016$ (i.e., the social planner is rather far-sighted), then it is always optimal to converge to an equilibrium with low numbers of infected and noninfected IDUs by using rather high levels of treatment; for $r > 0.125$ (rather myopic social planner), optimal convergence is always to a stationary state with zero treatment spending and hence high numbers of IDUs (both infected and noninfected).

4. Interpretation of the Results

The analysis of the dynamics has demonstrated that the long-run optimal behavior may depend crucially on the initial conditions both as concerns the total number of IDUs as well as concerns the number of infected IDUs relative to the total number of IDUs. If there are no infected IDUs at $t = 0$, the boundary solution $P = 0$ is the optimal solution in the long run. This result seems to be obvious because, in this case, there is no HCV infection and there are only costs due to the consumption of drugs. Therefore, this boundary solution exists also independently of the additional costs caused by an IDU being infected, denoted by ν in our model. But it must be repeated that this long-run solution is feasible only if the cure rate due to methadone treatment, the exit rate of drug users due to treatment, and if the costs of an noninfected IDU are relatively high (as to this point see also the discussion in Section 3.2). However, there may exist also an interior long-run solution which goes along with a small value of the ratio of infected IDUs relative to the total number of IDUs. But such a long-run solution can be obtained only by a high intervention, that is, by a large amount for methadone spending (see the following remarks).

Besides this boundary solution, one or two saddle point stable interior long-run optima may exist depending on the cost of a new infection. When there are two, the first interior optimum implies a low number of infected IDUs and a low total number of IDUs. This solution is generated by funding a high ratio of methadone treatment "slots" relative to the total number of IDUs. This means that, in this situation, it is optimal to spend a lot for methadone programs in order to reduce the number of both infected and noninfected IDUs. In Figure 1, the dashed lines in the grey-shaded area show paths of optimal policies with intensive methadone treatment. However, it is only optimal to converge to this solution if the initial number of IDUs and infected IDUs is relatively small. Further, this solution exists only if the cost per HCV infection exceeds a certain value which is about 2873 (i.e. for $\nu \geq 2873$) in our example. This is plausible because, with lower costs of HCV, it cannot be optimal to spend a lot for methadone projects in order to reduce HCV infections.

The second interior solution implies a high number of total and infected IDUs but a small amount of methadone treatment. In this situation, it is economically

optimal to spend less for methadone programs and to accept a high number of addicts. If the initial values of IDUs and infected IDUs are relatively large, it is optimal to converge to this equilibrium in the long run. In this case, spending for methadone programs is small and we may speak of "accommodation" policies. This holds although it would be possible to control the system so that it converges to E^2 as the dashed lines starting in the white area in Figure 1 show. However, such policies are not cost optimal. Loosely speaking, it is too expensive to reduce the number of IDUs when the initial number is high. The dotted lines in the white area in Figure 1 show optimal paths of prototypical accommodation policies. Only if the initial values $P(0)$ and $N(0)$ lie on the curve which separates the two areas is the planner indifferent ion whether the equilibrium E^2 or E^4 is obtained in the long run. This holds because, in this case, convergence to E^2 yields the same value for (7) as convergence to E^4.

This shows that, for certain values of the cost associated with a new infection, a threshold exists so that the initial conditions are crucial as to which equilibrium the system converges to in the long run. However, if the costs of a new infection are extremely high (i.e. higher than 13253.6 in our example), no threshold exists and it is always optimal to spend a lot for methadone and to reduce the number of infected IDUs. If the costs of a new infection are lower than 9355.4, an accommodation policy turns out always to be optimal leading to a large number of infected IDUs in the long run, although other equilibria exist. In both these cases, the long-run outcome is independent of the initial conditions. When the cost of a new infection lies between these two boundaries, then the initial conditions matter and determine the long-run outcome. If the costs are smaller than 2873, the accomodation solution is the only feasible long-run outcome, because no other interior equilibrium with real values exists.

The values of the cost of a new infection for which a DNS curve exists depend also on the infectivity. The higher the infectivity, the larger the interval is for which a threshold exists. This means that, with a high infectivity, a threshold can exist even for lower or higher values of the cost of a new HCV infection. From an economic point of view, this means that the optimization problem for the planner is more likely to become complex when the infectivity is high. With a high infectivity, the planner must be aware that the optimal long-run solution may depend crucially on the initial number of infected and noninfected IDUs.

5. Conclusions

This paper has demonstrated that the long-run stationary solution to an optimal control problem, which is to alleviate the negative economic consequences of drug addiction, may not be unique. Using empirically plausible values, we could demonstrate that the optimal spending for drug substitution programs may

depend crucially on initial conditions as concerns the number of drug users and the number of infected users. Thus, it may be that the optimal long-run solution is characterized by a relatively small amount spent for drug substitution programs and by a high number of drug addicts or vice versa.

We want to stress that our model can be seen as an exercise in cost-benefit analysis because it compares the cost of a certain health program to its benefits. Given the result of our analysis, it becomes clear that this task may be a complex one. So, the policy maker should be aware of the fact that there can be more than one optimal long-run solution depending on the starting values. The conditions determining which long-run solution is optimal in our model can be also of help for the policy maker to find an optimum in reality. This holds because our study gives hints as to when it is optimal to spend a lot for methadone projects and when the reverse holds.

There are several opportunities for refinements and further work. Notably, the initiation rate or inflow of new users could be made a function of the current number of users (cf. Refs. 14–15). Alternatively, the policy maker could jointly optimize spending on both treatment and other drug control interventions such as enforcement (cf. Ref. 15). Finally, one could imagine an expanded state space that explicitly tracked the number of users who are infected with either or both of two or more types of infections.

References

1. TAMI, L. M., WOODY, G. E., JUDAY, T., and KLEBER, H. D., *The Economic Costs of Heroin Addiction in the United States*, Drug and Alcohol Dependence, Vol. 61, pp. 195–206, 2001.
2. KOPP, P., RUMEAU-PICHON, C., and LE PEN, C., *The Financial Impact of Maintenance Treatment in Heroin Addictive Behavior: The Case of Subutex*, Revúe d'Epidemiologie et de Santé Publique, Vol. 48, pp. 256–270, 2000.
3. GAVRILA, C., POLLACK, H. A., CAULKINS, J. P., KORT, P. M., FEICHTINGER, G., and TRAGLER, G., *Optimal Control of Harm Reduction in Preventing Blood-Borne Diseases among Drug Users*, Research Report 255, Institute for Econometrics, Operations Research, and Systems Theory, Vienna University of Technology, 2001.
4. POLLACK, H. A., *Cost-Effectiveness of Harm Reduction in Preventing Hepatitis C among Injection Drug Users*, Medical Decision Making, Vol. 21, pp. 357–367, 2001.
5. KURZ, M., *The General Instability of a Class of Competitive Growth Processes*, Review of Economic Studies, Vol. 35, pp. 155–174, 1968.
6. WOLFRAM RESEARCH, *Mathematica: A System for Doing Mathematics by Computer, Version 2.0*, Champaign, Illinois, 1991.
7. KAPLAN, E. H., *Needles That Kill: Modeling Human Immunodeficiency Virus Transmission via Shared Needle Injection Equipment in Shooting Galleries*, Reviews of Infectious Diseases, Vol. 11, pp. 289–298, 1989.

8. HOMAS, D. L., VLAHOV, D., SOLOMON, L., COHN, S., TAYLOR, E., and GARFEIN, R., *Correlates of Hepatitis C Virus Infection among Injection Drug Users*, Medicine, Vol. 74, pp. 212–220, 1995.
9. POLLACK, H. A., *The Cost-Effectiveness of Methadone in Preventing Blood-Borne Disease: A Comparison of HIV and Hepatitis C*, Working Paper, University of Michigan, 2000.
10. POSTMA, M. J., WIESSING, L. G., and JAGER, J. C., *Pharmaco-Economics of Drug Addiction: Estimating the Costs of Hepatitis C Virus, Hepatitis B Virus, and Human Immunodeficiency Virus Infection among Injecting Drug Users in Member States of the European Union*, Bulletin on Narcotics, Vol. 53, pp. 79–89, 2001.
11. GOLD, M. R., SIEGEL, J. E., RUSSELL, L. B., and WEINSTEIN, M. C., *Cost-Effectiveness in Health and Medicine*, Oxford University Press, New York, NY, 1996.
12. DECHERT, W. D., and NISHIMURA, K., *A Complete Characterization of Optimal Growth Paths in an Aggregated Model with a Nonconcave Production Function*, Journal of Economic Theory, Vol. 31, pp. 332–354, 1983.
13. SKIBA, A. K., *Optimal Growth with a Convex-Concave Production Function*, Econometrica, Vol. 46, pp. 527–540, 1978.
14. BEHRENS, D. A., CAULKINS, J. P., TRAGLER, G., and FEICHTINGER, G., *Optimal Control of Drug Epidemics: Prevent and Treat, But Not at the Same Time?*, Management Science, Vol. 46, pp. 333–347, 2000.
15. TRAGLER, G., CAULKINS, J. P., and FEICHTINGER, G., *Optimal Dynamic Allocation of Treatment and Enforcement in Illicit Drug Control*, Operations Research, Vol. 49, pp. 352–362, 2001.